Chapter 8: Test of Statistical Hypotheses

8.2 Tests of the Equality of Two Means

Let independent random variables X and Y have normal distributions $N(\mu_X, \sigma_X^2)$ and $N(\mu_Y, \sigma_Y^2)$, respectively. There are times when we are interested in testing whether the distributions of X and Y are the same. So if the assumption of normality is valid, we would be interested in testing whether the **two means are equal**.

Let X and Y are independent and normally distributed.

Hypothesis test for the difference between means ($\mu_1 - \mu_2$):

1. Write the Null and Alternative Hypothesis

2. Find the test statistic

3. Find the p-value/ Critical region

4. Make the decision

   (a) Technical:

   (b) English:

Example 1. A botanist is interested in comparing the growth response of dwarf pea stems against two different levels of the hormone indoleacetic acid (IAA). Using 16-day-old pea plants, the botanist obtains 5-mm sections and floats these sections on solutions with different hormone concentrations to observe the effect of the hormone on the growth of the pea stem. Let X and Y denote, respectively, the independent growths that can be attributed to the hormone during the first 26 hours after sectioning for $(0.5)(10)^{-4}$ and 10^{-4} levels of concentration of IAA.

The botanist measured the growths of pea stem segments, in millimeters, for $n = 11$ observations of X: 0.8, 1.8, 1.0, 0.1, 0.9, 1.7, 1.0, 1.4, 0.9, 1.2, 0.5. She did the same with $m = 13$ observations of Y: 1.0, 0.8, 1.6, 2.6, 1.3, 1.1, 2.4, 1.8, 2.5, 1.4, 1.9, 2.0, 1.2.

Perform a hypothesis test.

Example 2. A product is packaged by a machine with 24 filler heads numbered 1 to 24, with the odd-numbered heads on one side of the machine and the even on the other side. Let X and Y equal the fill weights in grams when a package is filled by an odd-numbered head and an even-numbered head, respectively. Assume that the distributions of X and Y are $N(\mu_X, \sigma^2)$ and $N(\mu_Y, \sigma^2))$, respectively, and that X and Y are independent.

For the $n = 12$ observations of X, $\bar{x} = 1076.75$ and $s_x^2 = 29.30$.

For the $m = 12$ observations of Y, $\bar{y} = 1072.33$ and $s_y^2 = 26.24$.

Perform a hypothesis test to check whether means are not equal.

modified tests

- If we are able to assume that we know the variances of X and Y, then the appropriate test statistic to use for testing $H_0 : \mu_X = \mu_Y$ is

- Second, if the variances are unknown and the sample sizes are large, *Example 3.* The target thickness for Fruit Flavored Gum and for Fruit Flavored Bubble Gum is 6.7 hundredths of an inch. Let the independent random variables X and Y equal the respective thicknesses of these gums in hundredths of an inch, and assume that their distributions are $N(\mu_X, \sigma_X^2)$ and $N(\mu_Y, \sigma_Y^2)$, respectively. Because bubble gum has more elasticity than regular gum, it seems as if it would be harder to roll it out to the correct thickness. To check this, they have used samples of sizes $n = 50$ and $m = 40$. Where $\bar{x} = 6.701$, $s_x = 0.108$, $\bar{y} = 6.841$ and $s_y = 0.155$. .

Perform a hypothesis test to check their claim.