Chapter 8: Test of Statistical Hypotheses

8.1 Tests about One Mean

Case I: One sample 4- step hypotheses test for μ (population mean) when σ^2 is <u>known</u>

Assume we sample from a normal distribution:

Step 1: Null and Alternative Hypotheses

Step 2: Test Statistic

A random sample is taken from the distribution and an observed sample mean, $\bar{x}$, that is close to μ_0 supports H_0. when σ is known, the test statistic could be defined by

Step 3: Critical Region or the P-value approach

The **critical regions**, at a significance level α, for the three respective alternative hypotheses would be

P-value Approach: P-value is commonly used for hypothesis testing to evaluate the similarity of difference in a data set. It can be defined in the following ways-

– It is the degree of confidence with which we can reject the null hypothesis (H_0)

– It is the measure of evidence that we have against the null hypothesis (H_0)

How to compute p-value?

Assume that H_0 is true.

P-value is the probability of observing a sample mean that is as or more extreme than the observed.

How to compute p-value for each type of hypothesis?

Step 4: Decision

Technical conclusion:

Non technical conclusion:

Example 1. Assume that the underlying distribution is normal with unknown mean μ but known variance $\sigma^2 = 100$. we want to test whether μ is greater than 60. Suppose that we obtain the observed sample mean $\bar{x} = 62.75$ based on n = 52 observations. Perform a hypothesis test.

Errors

- Type I error
- Type II error
- significance level:

Case II: One sample 4- step hypotheses test for μ (population mean) when σ^2 is <u>unknown</u>

Step 1: Null and Alternative Hypotheses

Step 2: Test Statistic

Step 3: P-value approach or Critical Region

The **critical regions**, at a significance level α, for the three respective alternative hypotheses would be

Step 4: Decision

Technical conclusion:

Non technical conclusion:

Example 2. Let X (in millimeters) equal the growth in 15 days of a tumor induced in a mouse. Assume that the distribution of X is $N(\mu, \sigma^2)$. Perform a hypothesis test to check whether mean is equal to 4 or not if we are given that n = 9, $\bar{x} = 4.3$, and s = 1.2, using n = 9 observations and a significance level of $\alpha = 0.10$.

Example 3. In attempting to control the strength of the wastes discharged into a nearby river, a paper firm has taken a number of measures. Members of the firm believe that they have reduced the oxygen-consuming power of their wastes from a previous mean μ of 500 (measured in parts per million of permanganate).

Case III: Paired T-test

Oftentimes, there is interest in comparing the means of two different distributions or populations.

consider two situations:

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For example, suppose that X and Y equal the resting pulse rate for a person before and after taking an eight week program in aerobic dance.

Example 4. Twenty-four girls in the 9th and 10th grades were put on an ultraheavy rope-jumping program. Someone thought that such a program would increase their speed in the 40-yard dash. Let W equal the difference in time to run the 40-yard dash—the "before-program time" minus the "after-program time." Assume that the distribution of W is approximately $N(\mu_W, \sigma_W^2)$.

The following data give the difference in time that it took each girl to run the 40-yard dash, with positive numbers indicating a faster time after the program:

0.28	0.01	0.13	0.33	-0.03	0.07	-0.18	-0.14
-0.33	0.01	0.22	0.29	-0.08	0.23	0.08	0.04
-0.30	-0.08	0.09	0.70	0.33	-0.34	0.50	0.06

For these data, $\bar{w} = 0.0788$ and $s_w = 0.2549$.

Two sided CI and Hypotheses tests:

The other way of looking at tests of hypotheses is through the consideration of confidence intervals, particularly for two-sided alternatives and the corresponding tests.

Consider the mouse data in example 2 and find a 90% CI for the data.