Chapter 7: Interval Estimation

7.2 Confidence Intervals for Difference of Two Means

Two Independent Samples (Four CIs will be discussed):

Suppose that we are interested in comparing the means of two normal distributions.

Consider two independent random samples:

Sample		
Sample Size		
Distribution		

Case I: σ_X^2 and σ_Y^2 are known:

Suppose, for now, that σ_X^2 and σ_Y^2 are known and the random samples are independent.

Therefore respective sample means $\bar{X}$ and $\bar{Y}$ are also independent and have distributions

Consequently, the distribution of $W = \bar{X} - \bar{Y} \sim$

Once the experiments have been performed and the means $\bar{x}$ and $\bar{y}$ computed, the interval

provides a $100(1-\alpha)\%$ confidence interval for $\mu_X - \mu_Y$.

Example 1. discussion, let $n = 15, m = 8, \bar{x} = 70.1, \bar{y} = 75.3, \sigma_x^2 = 60, \sigma_y^2 = 40$, and $1 - \alpha = 0.90$. Find a 90% CI for $\mu_X - \mu_Y$.

Case II: σ_X^2 and σ_Y^2 are unknown but, the sample sizes are large:

If the sample sizes are large and σ_X^2 and σ_Y^2 are **unknown**, we can replace σ_X^2 and σ_Y^2 with ______ and ______ where ______ and ______ are the values of the respective **unbiased estimates of the variances**. This means that

serves as an **approximate** $100(1-\alpha)\%$ confidence interval for $\mu_X - \mu_Y$.

Case III: σ_X^2 and σ_Y^2 are unknown but assumed equal. Also, the sample sizes are small:

Now consider the problem of constructing confidence intervals for the difference of the means of two normal distributions when the variances are unknown but the **sample sizes are small**.

This problem can be a difficult one. However, even in these cases, if we can assume common, but unknown, variances ____________.

Example 2. Suppose that scores on a standardized test in mathematics taken by students from large and small high schools are $N(\mu_X, \sigma^2)$ and $N(\mu_Y, \sigma^2)$ respectively, where σ^2 is unknown. If a random sample of $n = 9$ students from large high schools yielded $\bar{x} = 81.31, s_x^2 = 60.76$, and a random sample of $m = 15$ students from small high schools yielded $\bar{y} = 78.61, s_y^2 = 48.24$. Find a 95% CI for $\mu_X - \mu_Y$.

Case IV: σ_X^2 and σ_Y^2 are unknown but the ratio $\frac{\sigma_X^2}{\sigma_Y^2}$ is known. Also, the sample sizes are small:

When population variances are *unknown* but the ratio $\frac{\sigma_X^2}{\sigma_Y^2}$ is known. And sample sizes are *not large*, an approximate $100(1-\alpha)\%$ confidence interval for $\mu_X - \mu_Y$ is given by (Welch CI)

Two Dependent Samples (One CI will be discussed):

When X and Y are taken on the same subject, X and Y may be dependent random variables. Many times these are ______________ and ______________ measurements,

Example: weight before and after participating in a diet-and-exercise program.

Let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ be n pairs of dependent measurements. Let $D_i = X_i - Y_i$, $i = 1, 2, \ldots, n$. Suppose that $D_1, D_2, \ldots, D_n$ can be thought of as a random sample from $N(\mu_D, \sigma_D^2)$, where μ_D and σ_D^2 are the mean and standard deviation of each difference. To form a confidence interval for $\mu_X - \mu_Y$, use

where _____ and _____ are, respectively, the sample _____ and sample _____ of the _____ __________.
Thus, T is a t statistic with _____ degrees of freedom.

$100(1 - \alpha)\%$ confidence interval for $\mu_D = \mu_X - \mu_Y$ is given by

where _____ and _____ are the __________ mean and standard deviation of the sample of the D values.

Example 3. An experiment was conducted to compare people's reaction times to a red light versus a green light. When signaled with either the red or the green light, the subject was asked to hit a switch to turn off the light. When the switch was hit, a clock was turned off and the reaction time in seconds was recorded. Find a 95% CI for the difference between the reaction times to a red light versus a green light.

The following results give the reaction times for eight subjects:

Subject	Red (x)	Green (y)	
1	0.30	0.43	
2	0.23	0.32	
3	0.41	0.58	
4	0.53	0.46	
5	0.24	0.27	
6	0.36	0.41	
7	0.38	0.38	
8	0.51	0.61	